From Ben-Or/Tiwari to QD and Generalized Eigenvalues II

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Ben-Or/Tiwari algorithm (embedded with Berlekamp/Massey algorithm)

Determine the poles of

$$g(x) = \frac{1}{(x^8+5)(x-7)} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

- Compute the Taylor coefficients for $g(x)$ at $x = 0$:

$$c_0 = g(0) = \frac{-1}{35},\ c_1 = \frac{g^{(1)}(0)}{1!} = \frac{-1}{245},\ c_2 = \frac{g^{(2)}(0)}{2!} = \frac{-1}{1715},\ c_3 = \frac{g^{(3)}(0)}{3!} = \frac{-1}{12005},\ c_4 = \frac{g^{(4)}(0)}{4!} = \frac{-1}{84035}, \ldots.$$

sequence $c_0, c_1, c_2, \ldots$ is linearly generated

- Compute the reverse of the minimal generator $\Lambda^{\text{rev}}(x)$ by Berlekamp/Massey algorithm or solving a Hankel system.

$$\Lambda^{\text{rev}}(x) = 1 + \frac{1}{5}x^8 - \frac{1}{35}x^9 - \frac{1}{7}x = \frac{-(x-7)(x^8+5)}{35}$$

The QD algorithm

Interpolate $f(x) = (x-7)(x^8+5) = x^9 - 7x^8 + 5x - 35$

- Pick $p \neq 1, 0, -1$: say $p = 2$.
  Evaluate $a_i = f(p^i)$

$$a_0 = f(2^0) = -36,\ a_1 = f(2) = -1305,\ a_2 = f(2^2) = -196623,\ a_3 = f(2^3) = 16777221, \ldots$$

- Initialize:

$$e_0^{(n)} = 0, \quad n = 1, 2, \ldots; \qquad q_1^{(n)} = \frac{a_{n+1}}{a_n}, \quad n = 0, 1, 2, \ldots.$$

- Continue with:

$$e_m^{(n)} = q_m^{(n+1)} - q_m^{(n)} + e_{m-1}^{(n+1)}, \quad m = 1, 2, \ldots, \quad n = 0, 1, 2, \ldots; \qquad q_{m+1}^{(n)} = \frac{e_m^{(n+1)}}{e_m^{(n)}} q_m^{(n+1)}, \quad n = 0, 1, 2, \ldots.$$

$$\begin{array}{cccccccc}
 & q_1^{(0)} & & & & & & \\
0 & & e_1^{(0)} & & & & & \\
 & q_1^{(1)} & & q_2^{(0)} & & & & \\
0 & & e_1^{(1)} & & e_2^{(0)} & & & \\
 & q_1^{(2)} & & q_2^{(1)} & & q_3^{(0)} & & \\
0 & & e_1^{(2)} & & e_2^{(1)} & & e_3^{(0)} & \\
 & q_1^{(3)} & & q_2^{(2)} & & q_3^{(1)} & & \cdots \\
0 & & \downarrow & & \downarrow & & \downarrow & \\
 & \downarrow & 0 & \downarrow & 0 & \downarrow & 0 & \downarrow \\
 & 512 & & 256 & & 2 & & 1 \\
 & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
 & x^9 & & x^8 & & x & & 1
\end{array}$$

- Recover the coefficients in $f(x)$ by solving a Vandermonde system.