A Mean Field Model for Dimensioning an OBS switch with Partial Wavelength Conversion and Fiber Delay Lines

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***Abstract*—In this paper we introduce a mean field model to analyze an optical switch equipped with both wavelength converters (WCs) and fiber delay lines (FDLs) to resolve contention in OBS networks. Under some very general conditions, that is, a general burst size distribution and any Markovian burst arrival process at each wavelength, this model determines the minimum number of WCs required to achieve a zero loss rate as the number of wavelengths becomes large. The mean field result is exact as the number of wavelengths goes to infinity and turns out to be very accurate for systems with (a few) hundred wavelengths, commonly occurring when using wavelength division multiplexing (WDM). Moreover, we show that if the number of WCs is underdimensioned, (i) periodic system behavior may occur (with the period being the greatest common divisor of the burst lengths) and (ii) increasing the number of WCs may even worsen the loss rate under the often studied minimum horizon allocation policy (as opposed to the minimum gap policy). Finally, we further demonstrate that in terms of the loss rate, including (more) FDLs may have little or no effect on the number of WCs required to achieve a near-zero loss, especially for higher loads.**

I. Introduction

Optical burst switching (OBS) has been proposed as a solution to minimize the opto-electronic translations at the backbone network switches [1], [2]. As only the burst header requires this translation, the main part of the signal can be processed in the optical domain. In consequence, OBS enables the switches to catch up with the growing transmission capacity of the optical fibers, driven by wavelength division multiplexing (WDM). With WDM several signals can be sent at the same time using different wavelengths, increasing the fiber capacity by tens or hundreds. As the main part of the signal is processed in the optical domain, contention can be resolved using wavelength conversion or optical buffering. A wavelength converter allows an incoming burst to use a different wavelength for transmission if the one it used to enter the switch is unavailable. On the other hand, optical buffering is implemented using Fiber Delay Lines (FDLs) that allow an incoming packet to be delayed for a specific amount of time proportional to the length of each fiber.

a) Our Contribution: In this work we introduce a mean field model of an optical switch equipped with a pool of full-range wavelength converters and a set of FDLs per output port. The mean field model is exact when the number of wavelengths tends to infinity, while it is shown, via time consuming simulations, to be very accurate when compared to a finite system with a large number of wavelengths. This case is particularly relevant as WDM technology has increased the number of signals that a single fiber can carry to more than a hundred. Our model allows a general burst size distribution while the burst arrival process at each wavelength is modeled as a Markovian arrival process (MAP) [3]. This process is able to represent general correlated inter-arrival times. In order to select a wavelength for a specific incoming burst we consider two different allocation policies, the minimum horizon and minimum gap policies, both explained in Section II.

As the loss rate in an Erlang loss model decreases to zero as the number of servers becomes large, it is clear that a near-zero loss rate can be realized for WDM links with hundreds of wavelengths provided that there are plenty of wavelength converters (WCs) available. On the other hand, as switches with a high number of WCs is not very cost effective, limiting their numbers is important. Therefore, in switches with partial wavelength conversion, one typically has only $C = \sigma W$ converters, with $\sigma \in (0, 1)$ and W the number of wavelengths. Some important questions that arise are: (i) how to determine σ to achieve a near-zero loss and (ii) how is σ affected by the presence of FDLs. The main *new* insights gathered from the mean field model can be summarized as follows:

1) The mean field model allows us to determine σ in the general setting defined above (using only a single run).
2) If the number of WCs is underdimensioned, meaning σ is selected too small, periodic system behavior may occur, which is a very unwanted effect in any system. The period seems to be equal the greatest common divisor of the burst lengths.
3) Moreover, if the number of WCs is too small, increasing the number of WCs may even worsen the loss rate under

the minimum horizon policy (which aims at minimizing the burst delays). This is not the case when the minimum gap policy is used.

4) The number of FDLs in the system may have little or no effect on the required σ, meaning if the number of WCs is sufficiently large, there might be no use in incorporating FDLs (as far as the loss rate is concerned).
5) Even if the number of WCs is insufficient, increasing the number of FDLs may not improve the loss rate. Moreover, the load tends to decrease the use of incorporating FDL buffers.

To the best of our knowledge, each of these conclusions is novel and of significant importance when designing optical switches with partial wavelength conversion and fiber delay lines.

b) Related work: In previous studies analytical models have been used to evaluate the effect of wavelength conversion on a bufferless switch [4], [5], and to examine the performance of a switch equipped with FDLs but without converters [6]–[8]. The analysis of a switch including both solutions turns out to be more complex since the multidimensional nature of a multi-wavelength switch has to be combined with the special queuing behavior of the optical buffer. The interaction of both wavelength conversion and FDLs has been analyzed by means of simulation models in [9]–[11]. Additionally, an approximation for the multi-wavelength case based on a single-wavelength model was presented in [12]. It is shown to work well for fixed packet size, few wavelengths and a specific allocation policy. In these studies, as well as in the present paper, the converters are assumed to have full-range conversion, i.e., a burst can be converted to any wavelength. The case where the bursts can only be converted to a restricted set of wavelengths has been treated in [13]–[16].

This paper is organized as follows: in Section II we present the characteristics of the switch under analysis and the wavelength allocation policies; Section III describes the mean field model in detail, while Section IV compares the results of the model with results from the simulation of a finite system. This section also analyzes the effect of the allocation policies and of various parameters on the performance of the switch.

II. The Optical Switch

In this section we describe the operation and main features of the optical switch, the wavelength allocation policies and some modeling issues relevant for the description of the switch. In this and the next sections we use the terms packet and burst interchangeably. The optical switch under analysis, shown in Figure 1, is made of a number of input/output ports, each one connected to a fiber with W wavelengths. The switch works in a synchronous manner, where the time is divided in equally-spaced slots and the state of the switch is observed at slot boundaries. The synchronous operation, as opposed to the asynchronous case, makes the switching matrix design simpler but requires packet synchronization and alignment [4], [10].

The arrival process at each wavelength is modeled as a MAP [3] characterized by the set of $m \times m$ matrices $\{B_0, B_1, \ldots, B_{L_{\max}}\}$, where $L_{\max}$ is the maximum packet length. The MAP is driven by an underlying Markov chain with transition matrix $B = \sum_{k=0}^{L_{\max}} B_k$. For $k \geq 1$, the (i, j) entry of the matrix B_k is the probability that a packet of size k arrives and the underlying Markov chain makes a transition from i to j. Correspondingly, B_0 contains the transition probabilities of the underlying chain involving no arrivals. The class of MAP processes has been used to model the arrival process at a bufferless optical switch [4]. It includes many well-known processes as special cases, e.g., the discrete-time versions of the Poisson process, interrupted Poisson process (IPP), Markov modulated Poisson process (MMPP), etc. When a burst arrives it is switched to the corresponding output port using its own wavelength, called home wavelength. If the home wavelength is available for transmission in the output port, the burst starts transmission immediately. If the wavelength is already transmitting another burst or has scheduled the transmission of a burst waiting in the FDL, the new packet is buffered using the FDL. In case the FDL has no available buffering capacity in that wavelength, the incoming burst is converted to a different wavelength using one of the available converters. If there are no idle converters or no wavelengths with available buffering capacity, the burst must be dropped. Thus, to resolve contention the switch first tries to buffer the signal and only if this is not possible it tries to convert it, aiming to minimize the converter usage, as the *minConv* strategy in [11].

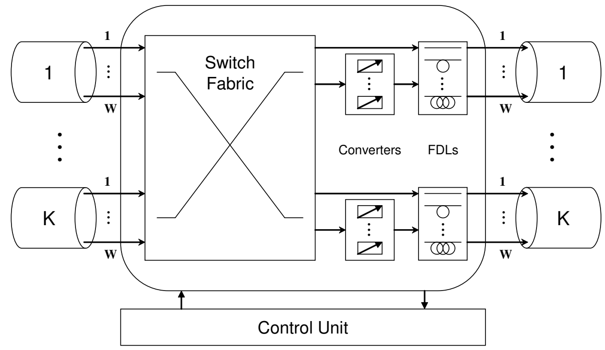


Figure 1. Optical switch with K input/output ports, W wavelengths, converters and FDLs

To analyze the performance of the switch we can focus on a single output port as the incoming traffic is assumed to be uniformly distributed among the output ports. To describe the state of one of these ports we consider two types of objects: wavelengths and converters. The state of a single wavelength is described by the scheduling horizon, which is the time until all the packets already scheduled for transmission in that wavelength have left the switch. If the horizon is equal to 0 and a packet of size L arrives, it can start transmission immediately and the horizon increases to L. On the other hand, if the incoming burst finds a horizon equal to h, it will experience a delay of at least h units before actual transmission. As the buffering is carried out by a set of N FDLs, the possible delay a packet can experience depends on the length of these delay lines. Here we assume the N FDLs have linearly growing length with granularity D, i.e., the first line provides a delay of D time slots, the delay in the second is equal to $2D$, and the last line delays the packet for ND slots. With this setup an incoming packet that observes a scheduling horizon equal to h has to wait for $D\left\lceil \frac{h}{D} \right\rceil$ slots, if $h \leq ND$. If the packet is of size L the new value of the horizon is $D\left\lceil \frac{h}{D} \right\rceil + L$. Notice, in this particular case the wavelength remains unused for a length of $D\left\lceil \frac{h}{D} \right\rceil - h$ just prior to the packet transmission, we refer to this as a *gap*. If h is greater than ND the packet cannot be buffered in the FDL using the same wavelength and it must be reallocated in another wavelength with horizon less than or equal to ND.

A packet that cannot be buffered in its home wavelength, called an *extra-packet*, can be reallocated if there is both a wavelength with scheduling horizon no greater than ND and an available converter. Hence, it is necessary to check the state of all the wavelengths and the converters. There are C converters per output port and the state of a single converter is also described by its scheduling horizon. In this case the converter has no buffering capacity, therefore its horizon reduces to the time required by the packet already in service to be completely converted to the other wavelength. Then, if an extra-packet of size L finds an available converter (and there is a wavelength with available buffering capacity) the horizon of the selected converter changes its value from 0 to L. Naturally, when this conversion occurs the horizon of the wavelength that receives the burst increases its value as described previously. An important assumption is that each wavelength with available buffering capacity can only receive one extra-packet during one slot, even if it has enough free FDLs to receive more than one additional packet. Removing this assumption would complicate both the possible set of wavelength allocation policies and its corresponding modeling aspects. The number of converters C per output port is determined as a fraction of the number of wavelengths W, i.e., $C = \sigma W$, where σ is the conversion ratio. If $\sigma = 0$ (resp. $\sigma = 1$) the switch is said to have null (resp. full) conversion. Here we assume that σ takes values between 0 and 1, which is called partial conversion. If an extra-packet finds an idle converter it has to choose a wavelength among those with horizon less than or equal to ND. This selection can be made using two different allocation policies: *minimum horizon*, which selects the wavelength with the minimum scheduling horizon; and *minimum gap*, which selects the wavelength with a horizon such that the allocation of a new packet generates a gap of minimum value. Recall, the gap is the difference between the horizon observed by an incoming packet and the actual delay that a packet assigned to the wavelength must face.

To model the evolution of the switch in a single slot we consider the following order of events: first, the busy wavelengths (resp. converters) transmit (resp. translate) part of the packet in service, reducing their horizons by one. Second, a new packet may arrive at each wavelength with a probability related to the phase of its arrival process; the packet is buffered if there is space available in its home wavelength, otherwise it becomes part of the set of extra-packets. Third, the extra-packets are converted to a different wavelength with available buffering capacity. Any extra-packet that does not find an available converter or a wavelength with buffering capacity must be dropped. The probability that a packet is dropped is called *loss probability* and is considered the main measure of performance.

III. The Mean Field model

Our model is based on a general result for a system of interacting objects introduced in [17]. In this case, the system consists of two types of objects: wavelengths and converters. To describe the evolution of the system during a time slot we start with the state of the objects at the beginning of the time slot. Then we determine the transition matrices that describe the state transitions at each of the three steps: transmission, arrivals and reallocation. The matrices associated to these steps are S_k, A_k and Q_k, respectively, where the subscript k may be equal to w or c depending on whether the matrix describes the transition of a wavelength or a converter. These matrices are then used to build a complete description of the evolution of the switch at each time slot. At the beginning of slot t (before packet transmission) the state of a single wavelength can be described by the tuple $\{(H(t), J(t)), t \geq 0\}$, with $H(t)$ the scheduling horizon of the wavelength and $J(t)$ the phase of its arrival process. Its state space is the set $\{(i,j) | 0 \leq i \leq ND + L_{\max}, 1 \leq j \leq m\}$. Similarly, a converter can be described by its scheduling horizon $\{(\bar{H}(t), t \geq 0\}$ with state space $\{i | 0 \leq i \leq L_{\max}\}$. We now define the evolution matrices for each of the three steps.

A. Step 1, packet transmission

In the first step (**S1**) the horizon of each busy wavelength and each busy converter is reduced by one, as they transmit (translate) part of the scheduled packets. Let T_k be the $(k+1) \times k$ matrix with entries

$$[T_k]_{ij} = \begin{cases} 1, & i = j = 1 \\ 1, & j = i-1, i = 2, \ldots, k+1 \\ 0, & \text{otherwise.} \end{cases}$$

Then the evolution of a single wavelength in **S1** is given by the transition matrix $S_w = T_{ND+L_{\max}} \otimes I_m$, where I_k is the identity matrix of size k and $\otimes$ denotes the Kronecker product. This product shows that packet transmissions affect the horizon value but not the phase of the arrival process. Accordingly, the matrix $S_c = T_{L_{\max}}$ contains the transition probabilities for a converter in **S1**.

B. Step 2, packet arrivals

The arrival of packets during the second step (**S2**) has no influence in the state of a converter; therefore, its transition matrix in this step is given by $A_c = I_{L_{\max}}$. Similarly, the matrix A_w describes the transition of a single wavelength in **S2**, but its definition is more involved. If the wavelength has a horizon less than or equal to ND after **S1**, it can accept any incoming packet. On the other hand, if the scheduling horizon is greater than ND and a packet arrives, it cannot be buffered and becomes part of the extra-packets. To keep track of the size of the possibly empty set of extra-packets, the horizon and the phase of the arrival process, we separate the resulting state space after **S2** into two sets. The first set is $\{(i,j)|0 \leq i \leq ND+L_{\max}, 1 \leq j \leq m\}$, which captures two cases: first, the horizon was less than or equal to ND after **S1**, and the transition in **S2** results in a horizon equal to i and a phase of the arrival process equal to j; second, the horizon was greater than ND but no packet is received. In this first set the wavelength holds zero extra-packets. The second set is $\{(ND + L_{\max} + (i-1)L_{\max} + k, j)|1 \leq i \leq L_{\max} - 1, 1 \leq k \leq L_{\max}, 1 \leq j \leq m\}$, considering the case when the horizon was equal to $ND+i$ after **S1** and the arrival process (during **S2**) generates a packet of size k and makes a transition to phase j. Therefore, the transition matrix A_w is of size $m(ND + L_{\max}) \times m(ND + L_{\max}^2 + 1)$ (as the maximum horizon value after step **S1** is at most $ND + L_{\max} - 1$).

To explicitly describe the matrix A_w we partition the state space before and after **S2** in levels, where the level k is the set of states $\{(k,j)|1 \leq j \leq m\}$. The matrix $A_w^{\{k,k'\}}$ contains the transition probabilities from level k to level k', for $0 \leq k \leq ND+L_{\max}-1$ and $0 \leq k' \leq ND+L_{\max}^2$. As the arrivals that find the wavelength idle can start transmission without any delay, the transitions from level 0 are given by $A_w^{\{0,k\}} = B_k$, for $0 \leq k \leq L_{\max}$. Also, the horizon is not modified if no packet arrives, then $A_w^{\{k,k\}} = B_0$, for $1 \leq k \leq ND + L_{\max} - 1$. On the other hand, if an incoming packet of size l finds a scheduling horizon between 1 and ND it must be buffered, affecting the horizon according to the transition probabilities given by $A_w^{\{k,k'\}} = B_l$, for $1 \leq k \leq ND$, $k' = D\left\lceil \frac{k}{D} \right\rceil + l$ and $1 \leq l \leq L_{\max}$. Finally, if a packet of size l arrives and the horizon is greater than ND, a conversion is required, making the transitions from this set of states equal to

$$A_w^{\{ND+k,ND+L_{\max}+k'\}} = B_l,$$

for $1 \leq k \leq L_{\max} - 1$, $k' = (k-1)L_{\max} + l$ and $1 \leq l \leq L_{\max}$. This completes the description of the transition matrices at **S2**.

C. Step 3, packet conversion and reallocation

In this step (**S3**) the extra-packets that arrived in the previous step are reallocated using the available converters. To determine the evolution of a *single* wavelength or converter it is necessary to consider the state of the whole system (W wavelengths and C converters). It is important to stress however that we do not need to determine the joint evolution of multiple wavelengths or converters for the mean field result to apply. Let $w_i(t)$ be the $1 \times m$ vector whose j-th entry contains the number of wavelengths holding no extra-packets with horizon equal to i and phase of the arrival process equal to j after **S2**, for $0 \leq i \leq ND+L_{\max}$ and $1 \leq j \leq m$. Additionally, let the j-th entry of the $1 \times m$ vector $w_{ND+iL_{\max}+k}(t)$ be the number of wavelengths at time t with horizon equal to $ND + i$ after **S1** that receive a packet of size k in **S2**, after which the phase of the arrival process is equal to j, for $1 \leq i \leq L_{\max} - 1$, $1 \leq k \leq L_{\max}$ and $1 \leq j \leq m$. The vector $M^{W,(w)}(t) = \frac{1}{W}[w_0(t), \ldots, w_{ND+L_{\max}^2}(t)]$ describes the state of all the wavelengths at time t before **S3** as fractions of the total number of wavelengths W. Analogously, let $c_i(t)$ be the number of converters with horizon equal to i at time t before **S3**, for $i = 0, \ldots, L_{max}$. The state of the converters at time t, as a fraction of the total number of converters, is therefore contained in the vector $M^{W,(c)}(t) = \frac{1}{C}[c_0(t), \ldots, c_{L_{\max}}(t)]$. The superscript W indicates that the system is composed of W wavelengths and $C = \sigma W$ converters.

The state of the complete system at time t can be described by the vector

$$M^W(t) = \left[\frac{1}{1+\sigma} M^{W,(w)}(t),\ \frac{\sigma}{1+\sigma} M^{W,(c)}(t)\right],$$

which is called the occupancy vector and contains the fraction of objects in each state, including both wavelengths and converters. The weights $\frac{1}{1+\sigma}$ and $\frac{\sigma}{1+\sigma}$ are the proportion of wavelengths and converters, respectively, in relation to the total number of objects. Based on this vector, we can define the matrices $Q_w(M^W(t))$ and $Q_c(M^W(t))$, which contain the transition probabilities in **S3** under the *minimum horizon* policy for wavelengths and converters, respectively. The matrices $\bar{Q}_w(M^W(t))$ and $\bar{Q}_c(M^W(t))$ contain similar information for the *minimum gap* policy. However, to specify these matrices it is necessary to first determine the number and size of the extra-packets that can actually be converted, regardless the wavelength allocation policy.

Let $d_i(M^W(t))$ be the number of extra-packets of size i, for $1 \leq i \leq L_{\max}$, which is given by

$$d_i(M^W(t)) = \sum_{k=1}^{L_{\max}-1} w_{ND+kL_{\max}+i}(t)\mathbf{1}_m,$$

where $\mathbf{1}_m$ is a column vector of size m with all its entries equal to one. Therefore, the total number of extra-packets is $d(M^W(t)) = \sum_{i=1}^{L_{\max}} d_i(M^W(t))$. Also, let $W_{ND}(M^W(t))$ be the number of wavelengths with horizon less than or equal to ND after **S2**, i.e., $W_{ND}(M^W(t)) = \sum_{i=0}^{ND} w_i(t)\mathbf{1}_m$. The number of extra-packets that can actually be converted ($\hat{d}(M^W(t))$) is given by the minimum of three quantities: the number of packets to convert, the number of wavelengths with available buffering capacity, and the number of available converters, i.e.,

$$\hat{d}(M^W(t)) = \min\{d(M^W(t)), W_{ND}(M^W(t)), c_0(t)\}.$$

Since each wavelength with available buffering capacity receives at most one extra-packet, $\hat{d}(M^W(t))$ is also the number of wavelengths that receive an extra-packet in **S3**. The selection of these $\hat{d}(M^W(t))$ wavelengths is done using the *minimum horizon* or *minimum gap* policies. Once a wavelength is chosen to receive an extra-packet, the selection of the packet is done randomly among the $d(M^W(t))$ extra-packets. This means that the probability that a selected wavelength receives a packet of size i, for $1 \leq i \leq L_{\max}$, is $p_i(M^W(t)) = \frac{d_i(M^W(t))}{d(M^W(t))}$. Relying on these definitions, the purpose of the following subsections is to determine the transition matrices for both wavelength allocation policies.

1) Minimum Horizon: To determine the wavelengths that, under the *minimum horizon* (*minH*) policy, will receive the $\hat{d}(M^W(t))$ extra-packets, we need to define the quantities $\alpha_i(M^W(t))$ as the number of wavelengths with horizon less than or equal to i after **S2**, i.e., $\alpha_i(M^W(t)) = \sum_{k=0}^{i} w_k(t)\mathbf{1}_m$, for $0 \le i \le ND$. As the extra-packets are assigned to the wavelengths with the smallest horizons, we need to find an $h(M^W(t))$ such that

$$\alpha_{h(M^W(t))-1} < \hat{d}(M^W(t)) \le \alpha_{h(M^W(t))}.$$

This means that the wavelengths with horizon strictly less than $h(M^W(t))$ receive one extra packet each, while those with a horizon strictly greater than $h(M^W(t))$ receive no extra-packets. The packets that cannot be accommodated in the wavelengths with horizons up to $h(M^W(t)) - 1$ are randomly assigned among the wavelengths with horizon equal to $h(M^W(t))$. Let $\theta(M^W(t))$ be the probability that a wavelength receives a packet in **S3** if its horizon is equal to $h(M^W(t))$. This is given by

$$\theta(M^W(t)) = \frac{\hat{d}(M^W(t)) - \alpha_{h(M^W(t))-1}}{w_{h(M^W(t))}(t)\mathbf{1}_m}.$$

Now we can define $r_i(M^W(t))$, the probability that a wavelength with horizon equal to i receives an extra-packet in **S3** under the *minH* policy, as

$$r_i(M^W(t)) = \begin{cases} 1, & 0 \le i < h(M^W(t)), \\ \theta(M^W(t)), & i = h(M^W(t)), \\ 0, & h(M^W(t)) < i \le ND. \end{cases}$$

Let $u_{ii'}(M^W(t))$ be the probability that a wavelength in state i after **S2** ends up with a horizon equal to i' after **S3**, for $0 \le i \le ND + L_{\max}^2$ and $0 \le i' \le ND + L_{\max}$. For clarity reasons we divide the definition of $u_{ii'}(M^W(t))$ in two parts. The first part refers to the possible reception of an extra-packet when the wavelength has a horizon less than or equal to ND after **S2**,

$$u_{ii'}(M^W(t)) = \begin{cases} 1 - r_i(M^W(t)), & 0 \le i = i' \le ND, \\ r_i(M^W(t))p_k(M^W(t)), & 0 \le i \le ND, \\ & i' = D\left\lceil \frac{i}{D} \right\rceil + k. \end{cases}$$

The second part considers the case of those wavelengths with horizon greater than ND after **S2**, which cannot accommodate an extra-packet and simply keep the same horizon they had,

$$u_{ND+i,ND+i'}(M^W(t)) = \begin{cases} 1, & 1 \le i = i' \le L_{\max}, \\ 1, & i'L_{\max} + 1 \le i \le (i'+1)L_{\max}, \\ & 1 \le i' \le L_{\max} - 1, \\ 0, & \text{otherwise}. \end{cases}$$

Here the second case is related to the wavelengths with horizon $ND + i'$ after **S1** that received an extra-packet in **S2** and now return to this original horizon as the extra-packet is either converted or dropped in step **S2** . Let $U(M^W(t))$ be the $(ND + L_{\max}^2 + 1) \times (ND + L_{\max} + 1)$ matrix with entries $u_{ii'}(M^W(t))$. Therefore, the transition matrix for a single wavelength during **S3** is $Q_w(M^W(t)) = U(M^W(t)) \otimes I_m$, making explicit that the allocation of extra-packets has no effect on the phase of the arrival process.

For the converters, only those with horizon equal to 0 may be affected during **S3** since these are used to translate the $\hat{d}(M^W(t))$ extra-packets. Let $b_i(M^W(t))$ be the probability that an idle converter receives a packet of size i in **S3**, for $1 \le i \le L_{\max}$. Also, let $b_0(M^W(t))$ be the probability that the converter remains idle. Clearly,

$$b_i(M^W(t)) = \begin{cases} \frac{c_0(t) - \hat{d}(M^W(t))}{c_0(t)}, & i = 0, \\ \frac{\hat{d}(M^W(t))}{c_0(t)} p_i(M^W(t)), & 1 \le i \le L_{\max}. \end{cases}$$

Therefore, the entries of the $L_{\max} \times (L_{\max} + 1)$ transition matrix for a single converter in **S3** can be defined as

$$[Q_c(M^W(t))]_{ij} = \begin{cases} b_j(M^W(t)), & i = 0,\ 0 \le j \le L_{\max} \\ 1, & 1 \le i = j \le L_{\max} - 1 \\ 0, & \text{otherwise}. \end{cases}$$

2) Minimum Gap: In this section we determine the matrices $\bar{Q}_w(M^W(t))$ and $\bar{Q}_c(M^W(t))$ to describe the evolution of the system during **S3** under the *minimum gap* (*minG*) policy. Since the wavelength allocation policy has no effect on the state of the converters, $\bar{Q}_c(M^W(t)) = Q_c(M^W(t))$. To specify the transition matrix for a single wavelength we start by defining the gap function $v(h) = D\left\lceil \frac{h}{D} \right\rceil - h$, which is the size of the gap created when assigning a packet to a wavelength with horizon h, for $0 \le h \le ND$. Now we can define $g_i(M^W(t))$ as the number of wavelengths with $v(\cdot) = i$, which is given by

$$g_i(M^W(t)) = \sum_{\{j|v(j)=i\}} w_j(t)\mathbf{1}_m, \quad 0 \le i \le D - 1.$$

In a similar way as in the previous section, we define $\gamma_i(M^W(t))$ as the number of wavelengths with $v(\cdot) \le i$, i.e., $\gamma_i(M^W(t)) = \sum_{j=0}^{i} g_j(M^W(t))$, for $0 \le i \le D - 1$. In this case we need to find an $x(M^W(t))$ such that

$$\gamma_{x(M^W(t))-1} < \hat{d}(M^W(t)) \le \gamma_{x(M^W(t))}.$$

Thus, $\gamma_{x(M^W(t))-1}$ extra-packets can be assigned to the wavelengths with $v(\cdot) < x(M^W(t))$. The packets that cannot be accommodated in these wavelengths are distributed among those with $v(\cdot) = x(M^W(t))$, while the rest of the wavelengths receive zero extra-packets. In this case, however, we use the *minH* policy to allocate these $\hat{d}(M^W(t)) - \gamma_{x(M^W(t))-1}$ extra-packets among the wavelengths with $v(\cdot) = x(M^W(t))$ (as opposed to randomly). Since the horizon h can be expressed as $h = D\left\lceil \frac{h}{D} \right\rceil - v(h)$ and the wavelengths that may receive a packet have $v(\cdot) = x(M^W(t))$, we only need to focus on $l(h) = \left\lceil \frac{h}{D} \right\rceil$, which takes values between 0 and N. Let $f_i(M^W(t))$ be the number of wavelengths with horizon h such that $v(h) = x(M^W(t))$ and $l(h) = i$, for $0 \le i \le N$. Also, let

$\phi_i = \sum_{j=0}^{i} f_j(M^W(t))$ be the number of wavelengths with horizon h such that $v(h) = x(M^W(t))$ and $l(h) \leq i$, for $0 \leq i \leq N$. We then need to find a $y(M^W(t))$ such that

$$\phi_{y(M^W(t))-1} < \hat{d}(M^W(t)) - \gamma_{x(M^W(t))-1} \leq \phi_{y(M^W(t))}.$$

Then, among the wavelengths with horizon h such that $v(h) = x(M^W(t))$, one extra-packet is assigned to the wavelengths with $l(h) < y(M^W(t))$, no extra-packet is assigned to those with $l(h) > y(M^W(t))$, and the rest of the extra-packets are randomly assigned among the wavelengths with $l(h) = y(M^W(t))$. Therefore, the probability that a wavelength with horizon h such that $v(h) = x(M^W(t))$ and $l(h) = y(M^W(t))$ receives an extra-packet during **S3** is

$$\eta(M^W(t)) = \frac{\hat{d}(M^W(t)) - \gamma_{x(M^W(t))-1} - \phi_{y(M^W(t))-1}}{f_{y(M^W(t))}(M^W(t))}.$$

Now we can define $\bar{r}_i(M^W(t))$ as the probability that a wavelength with horizon equal to i receives an extra-packet in **S3** under the *minG* policy, given by

$$\bar{r}_i(M^W(t)) = \begin{cases} 1, & 0 \leq v(i) < x(M^W(t)), \\ 1, & v(i) = x(M^W(t)), \\ & l(i) < y(M^W(t)), \\ \eta(M^W(t)), & v(i) = x(M^W(t)), \\ & l(i) = y(M^W(t)), \\ 0, & \text{otherwise.} \end{cases}$$

Based on these probabilities we can build a matrix $\bar{U}(M^W(t))$ in the same manner as the matrix $U(M^W(t))$ for the *minH* policy, but replacing the $r_i(M^W(t))$ by $\bar{r}_i(M^W(t))$, for $0 \leq i \leq ND$. Thus, the transition matrix of a wavelength in **S3** under the *minG* policy is $\bar{Q}_w(M^W(t)) = \bar{U}(M^W(t)) \otimes I_m$.

D. Computation of $M^W(t)$ for large W

In the previous sections we built the transition matrices related to each of the three main events (steps) in a slot, for wavelengths and converters separately. These matrices can be combined to describe the evolution of a single object as a discrete-time Markov chain (DTMC). We will observe the system just after **S2** and, therefore, the state at time t of the wavelengths (resp. converters) is described by the vector $w^W(t)$ (resp. $c^W(t)$). Since the order of the events is **S3**, **S1** and **S2**, the transition matrices of a single wavelength or converter under the *minH* policy are

$$K_k^W(M^W(t)) = Q_k(M^W(t))\, S_k\, A_k, \;\; k \in \{w, c\}.$$

Here the superscript W refers to the total number of wavelengths in the system. We now combine these two matrices into $K^W(M^W(t))$ to describe the evolution of a single object, which can be a wavelength or a converter, as a DTMC with two non-communicating classes

$$K^W(M^W(t)) = \begin{bmatrix} K_w^W(M^W(t)) & 0 \\ 0 & K_c^W(M^W(t)) \end{bmatrix}.$$

A similar construction can be made to determine the matrix $\bar{K}^W(M^W(t))$ for the *minG* policy.

We now consider the framework in [17] to compute $M^W(t)$ when W is large. The discussion is for the *minH* policy, but it applies *mutatis mutandis* for the *minG* policy. In [17] the authors show that, under some mild conditions, a system of interacting objects converges to its mean field when the number of objects is large. The mean field is a time-dependent deterministic system that can be used to approximate the behavior of a system with a large number of objects. The first condition for this result to hold is that the entries of the transition matrix of a single object $[K^W(M^W(t))]_{ij}$ converge uniformly to some $[K(M^W(t))]_{ij}$ on the set of all occupancy vectors when $W + C \to \infty$. In our model the transition matrix $K^W(M^W(t))$ is actually independent of the number of objects $W + C$. This can be seen by dividing all the quantities involved in the computation of the probabilities $u_{ii'}(M^W(t))$ and $b_i(M^W(t))$ by $W + C$. This means that $K(M^W(t)) = K^W(M^W(t))$. The second condition is that $[K(M^W(t))]_{ij}$ must be continuous in $M^W(t)$, which also holds for both allocation policies. Since both conditions are valid for the model described by the matrix $K(M^W(t))$, we can approximate the evolution of the system by means of the mean field, which is described by the vector $\mu(t)$, for $t \geq 0$. Let $\mu(t) = \left[\frac{1}{1+\sigma}\mu^{(w)}(t), \frac{\sigma}{1+\sigma}\mu^{(c)}(t)\right]$, for $t \geq 0$. We define the initial state of wavelengths and converters as $\mu^{(w)}(0) = [\pi_B, 0, \ldots, 0]$, where the $1 \times m$ vector π_B is the stationary probability distribution of the phase arrival process, and $\mu^{(c)}(0) = [1, 0, \ldots, 0]$. The initial distribution is independent of the number of objects and establishes that all the wavelengths and converters are idle at time 0. Now, let the mean field model evolve as $\mu(t+1) = \mu(t)K(\mu(t))$, then, by [17, Theorem 4.1], for any fixed time t, almost surely,

$$\lim_{W \to \infty} M^W(t) = \mu(t).$$

Using the mean field model we can compute the state of the system at time t by performing t vector-matrix multiplications, where the vector is of size $1 \times m(ND + L_{\max}^2 + L_{\max} + 1)$. We are particularly interested in the long-run behavior of the switch but the mean field model is time-dependent and gives no additional information about the steady-state behavior, if it exists. However, we have numerically observed that when the conversion ratio is large enough to prevent losses caused by the lack of available converters, the state of the system converges to a unique steady state. When the conversion ratio is not enough to avoid packet losses the system shows a stationary periodic behavior. The length of the period was observed to be the greatest common divisor of the possible packet sizes. Even though we do not provide a formal proof of this fact, the results presented in the next section, as well as many others not included here, support this observation. Let δ be the greatest common divisor of the possible packet sizes. As we do not know in advance if the conversion ratio is enough to prevent losses or not[1], we observe the system every δ time slots to check the difference in the entries of the state vector, and

[1] Actually, by running the mean field model once with $\sigma = 1$, we can determine the required σ value at once.

we let it evolve until this difference is less than $\epsilon = 10^{-10}$. For each of the δ steady states we compute the performance measures, as shown in the next section, and their average is the value of the steady-state performance measures.

E. Computation of the measures of performance

If time t corresponds to a steady state, then $d(M^W(t))$ is also the number of packets requiring conversion per slot in this steady state, which we call the spill *rate*. Similarly, $\hat{d}(M^W(t))$ is the conversion *rate*, and $d(M^W(t)) - \hat{d}(M^W(t))$ the loss *rate*. In a system with W wavelengths the total arrival rate is $W\lambda$, where λ is the arrival rate at each wavelength, given by $\lambda = \pi_B \sum_{k=1}^{L_{\max}} B_k \mathbf{1}_m = \pi_B(I_m - B_0)\mathbf{1}_m$. Therefore the spill probability p_{spill}, i.e., the probability that an incoming packet requires conversion, is given by $p_{\text{spill}} = \frac{d(M^W(t))}{W\lambda}$. Dividing the numerator and denominator by the number of objects $W + C$, we get

$$p_{\text{spill}} = \frac{\delta(M^W(t))}{\frac{\lambda}{1+\sigma}} = \frac{\sum_{i=1}^{L_{\max}} \sum_{k=1}^{L_{\max}-1} M^{W,(w)}_{ND+kL_{\max}+i}(t)\mathbf{1}_m}{\frac{\lambda}{1+\sigma}},$$

where $\delta(M^W(t)) = \frac{d(M^W(t))}{W+C}$ is independent of the number of objects. Likewise, we define $\hat{\delta}(M^W(t))$ as $\frac{\hat{d}(M^W(t))}{W+C}$, which allows us to define the conversion probability p_{conv} and the loss probability p_{loss} as

$$p_{\text{conv}} = \frac{\hat{\delta}(M^W(t))}{\frac{\lambda}{1+\sigma}}, \; p_{\text{loss}} = \frac{\delta(M^W(t)) - \hat{\delta}(M^W(t))}{\frac{\lambda}{1+\sigma}}.$$

IV. Results

In this section we first concentrate in the long-run behavior of the model, showing the periodic and non-periodic cases. Next, we compare the results of the mean field model with estimates from the simulation of a switch with a finite number of wavelengths. Finally, we analyze the effect of various parameters on the switch performance. In Figure 2 we illustrate the time-dependent behavior of the mean field model using the fraction of converters with horizon equal to 5, i.e., $\mu_5^{(c)}(t)$. The selection of this value is arbitrary as all the other entries in the state vector behave in a similar manner. To fix the arrival rate we use the load $\rho = \lambda\bar{L}$, where $\bar{L}$ is the expected value of the packet size. In this scenario the switch has $N = 3$ FDLs per output port, the load ρ is 0.8, the granularity is $D = 10$, the burst length equals 10, the inter-arrival times (IATs) follow a geometric distribution (meaning $B_0 = 1 - 0.8/10 = 0.92$ and $B_{10} = 0.8/10 = 0.08$), the policy is *minG* and the conversion ratio is between 0.1 and 0.3. As can be seen, when the conversion ratio is equal to 0.1 the state of the converters is highly variable and after a short warm-up period it adopts a periodic behavior. When the conversion ratio rises to 0.2 the warm-up period becomes longer and the state of the converters is clearly less variable, but the period is exactly the same and equal to the packet size, in this case 10 slots. Finally, if the conversion ratio is equal to 0.3 no losses are caused by lack of converters. In this case the warm-up period is even longer but

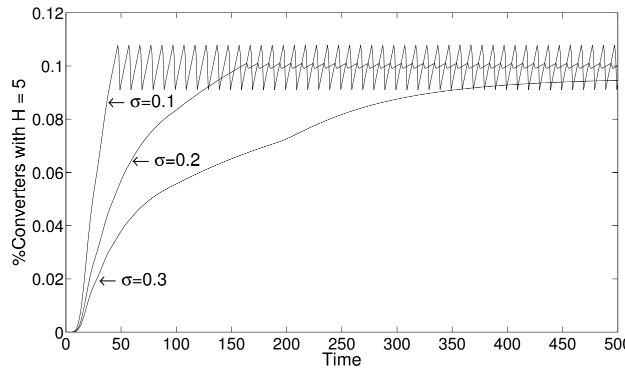
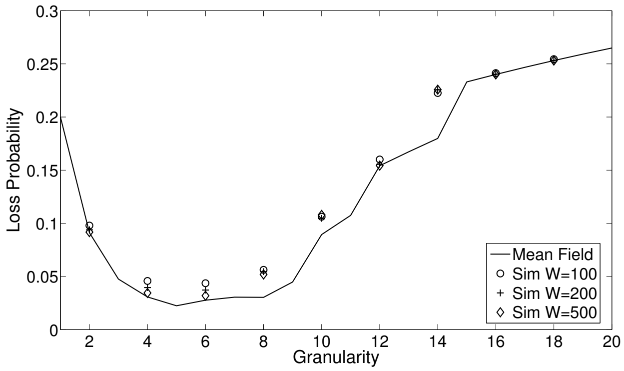


Figure 2. Time-dependent behavior of a switch with $N = 3$, $\rho = 0.8$, $D = 10$, geometric IATs and packet size equal to 10

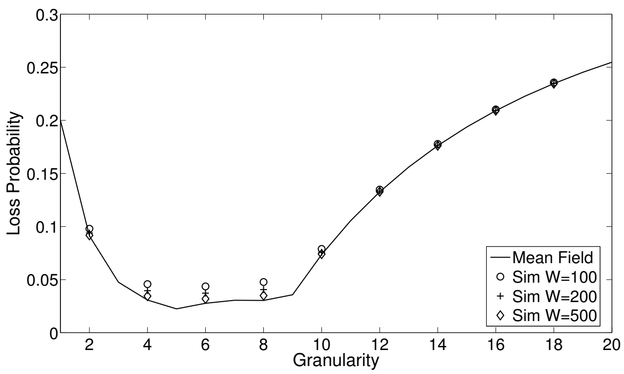
the system reaches a unique steady state. A similar behavior has been observed in all the experiments performed (including the simulations), with a periodic steady state and period equal to the greatest common divisor of the possible packet sizes. This periodic behavior arises when the conversion ratio is not enough to prevent packet losses. This is an important observation as it indicates that an underdimensioned number of WCs leads to a periodic system behavior. If there are plenty of converters to translate any extra-packet, the system converges to a unique steady state, as in Figure 2 for $\sigma = 0.3$.

A relevant issue for the mean field model is how it approximates the behavior of a finite system. Here we compare the results of the mean field model with results from simulation of a switch with 100, 200 and 500 wavelengths. The estimates from simulations have confidence intervals with half width less than 1% of the mean, obtained with the batch-means method. As can be expected, the simulations require long execution times to obtain a small confidence interval, particularly for a large number of wavelengths and small loss probabilities. Figure 3 shows how the performance of the finite system tends to that of the mean field model, getting closer as the number of wavelengths increases. In this scenario, as in many others, the convergence for the *minG* policy, shown in Figure 3(b), is smoother than for the *minH* policy, shown in Figure 3(a). This is useful since the *minG* policy tends to use the buffer capacity in a more efficient manner as shown further on.

We now compare the spill, conversion and loss probabilities for both allocation policies. In Figure 4 these three quantities are shown for a switch with $N = 3$ FDLs, granularity $D = 10$, load equal to 0.8, geometric arrivals and packet size with equally probable values 8 and 12. For both policies the conversion probability increases linearly with the number of converters up to a point from which it no longer increases. During the interval where this probability increases the converters are the bottleneck of the system, and therefore they are busy all the time. When the switch has enough converters to translate any extra-packet, i.e., when spill and conversion probabilities are equal, the switch no longer experiences losses due to the lack of converters. Notice, we can even determine the σ value where the loss rate becomes zero by running the mean field



(a) *minimum horizon* policy



(b) *minimum gap* policy

Figure 3. Mean field model vs. simulation for a switch with $N = 5$, $\rho = 0.8$, $\sigma = 10$, packet size equal to 10 and geometric IATs

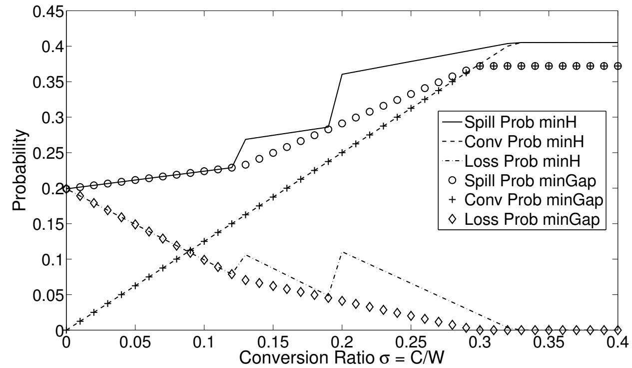


Figure 4. Comparison of policies for a switch with $N = 3$, $\rho = 0.8$, $D = 10$, geometric arrivals and packet size equal to $\{8, 12\}$

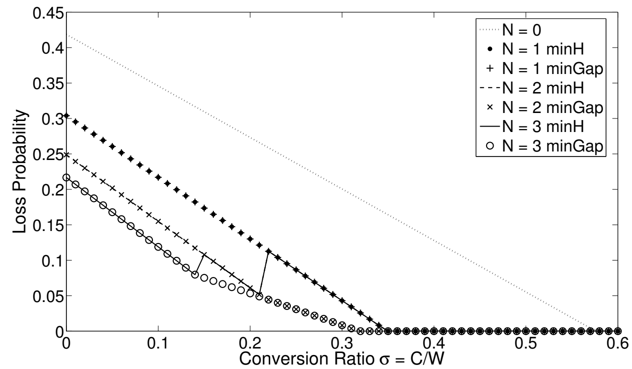


Figure 5. Comparison of policies for a switch with $\rho = 0.8$, $D = 10$, geometric arrivals and packet size equal to $\{5, 15\}$

model once with $\sigma = 1$ and noting the percentage of busy converters, solving the dimensioning problem of WCs in a single run. The *minG* policy requires a smaller conversion ratio to reach the point where spill and conversion probabilities are the same than the *minH* policy. Furthermore, from this point on the spill probability under *minH* is larger than under *minG*, confirning the well-known result that *minH* is less efficient in managing the buffering resources (FDLs). An observation that can be made from Figure 4, also found in Figure 5 as well as in many other experiments, is the existence of jumps in the spill and loss probabilities as a function of the conversion ratio, for the *minH* policy. These jumps are closely related to the discrete nature of the FDLs and the way the *minH* policy reallocates the extra-packets. As this policy selects the wavelengths with minimum horizon, the reallocated packets go first to the wavelengths with horizon 0 and, if the number of converted packets is larger than the number of wavelengths with horizon 0, the packets are sent to the wavelengths with horizon equal to 1. However, this allocation creates large gaps (of size $D - 1$) in the wavelengths that receive the converted packets. This implies that the gap size distribution is affected in a bad manner, reducing the capacity of the wavelengths and causing the spill probability to increase. Hence, the jump in the spill probability, and therefore in the loss probability, is caused by an increase in the conversion ratio that makes the system able to convert more packets than the wavelengths with horizon equal to 0 are able to admit. This jump can be seen in Figure 4 when σ goes from 0.12 to 0.13. The other jumps occur similarly when the conversion ratio goes from a value in which the reallocated packets can be handled by the wavelengths with horizon less than or equal to iD to a value in which they cannot, for $1 \leq i \leq N$. Notice that the number of jumps is at most equal to N but might be less than this value.

Another interesting issue for the design of an optical switch is the influence of the number of FDLs on the loss probability. Figure 5 shows the loss probability as a function of the conversion ratio, for a variable number of FDLs and both allocation policies. The packet size can be 5 or 15 with equal probability, the load is 0.8 and the granularity is 10. The effect of adding FDLs on the loss probability depends on the conversion ratio. If the conversion ratio is large enough, then adding more FDLs has *no effect*. However, the conversion ratio σ where the loss rate drops to zero does depend on N. For instance, in Figure 5, having $N = 1$ FDLs allows us to use significantly fewer WCs compared to having zero FDLs, while increasing N to 2 has a smaller effect, and an additional FDL has no effect (as a buffer capacity of $N = 2$ suffices with $C = 0.3W$ WCs). If σ is such that the switch has losses due to the lack of converters, then the addition of buffering capacity might reduce the losses substantially.

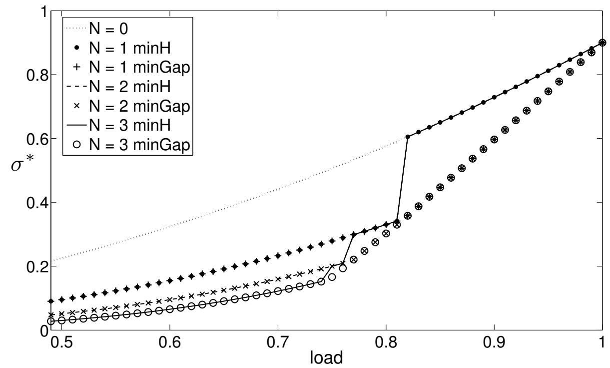


Figure 6. Comparison of policies for a switch with $D = 10$, geometric arrivals and packet size uniformly distributed between 5 and 15

However, adding an extra FDL might also have no effect at all, even if the switch presents losses. This is clear in Figure 5 for $\sigma = 0.25$, where the loss with two FDLs is lower than with one, but the addition of a third makes no difference.

As stated before, we can determine the value of σ at which the loss probability drops to zero, denoted σ^*, in a single run of the mean field model. In Figure 6 we illustrate how the load affects the value of σ^* for both policies. In this case the IATs follow a geometric distribution, the packet size is uniformly distributed between 5 and 15, and the granularity is 10. As expected, a higher load implies a larger σ^*. Also, for high loads the *minG* policy requires a smaller conversion ratio to achieve zero losses than the *minH* policy. In relation to the number of FDLs, it is clear that the addition of one FDL reduces the value of σ^* for the *minG* policy, but the effect of additional FDLs depends on the load. For high loads, there is no difference in having one or more FDLs, while for middle and low loads the addition of FDLs may reduce the value of σ^*. If the switch has enough converters to prevent losses and the load is one, the probability that a wavelength has horizon less than ND after **S1** is almost zero in steady state. When the load diminishes, the probability that the horizon is between $(N-1)D$ and $ND-1$ smoothly increases, but for values less than $(N-1)D$ it remains close to zero. To obtain a positive probability of having a wavelength with horizon less than $(N-1)D$ it is necessary for the load to go below a certain threshold, which in Figure 6 corresponds to 0.82. This behavior is independent of the value of N, explaining why the addition of more than one FDL has no effect in the conversion ratio required to achieve zero losses for loads over 0.82 in this scenario. Similar thresholds can be found for the values of the load required to have a positive probability that a wavelength has a horizon between $(i-1)D$ and $iD-1$, for $1 \leq i \leq N$. Hence, for loads above these thresholds having more than $N-i+1$ FDLs has no effect on σ^*. These thresholds coincide with the location of the jumps for the *minH* policy, but under this policy the probability of having a horizon less than ND is zero if $\sigma \geq \sigma^*$ and the load is greater than 0.82. If the load goes below this value, the probability of a horizon between $(N-1)D$ and $ND-1$ suddenly becomes positive and takes similar values to those of the *minG* policy. Therefore, both policies reach a similar σ^* at $\rho = 0.81$, but the *minG* policy does it in a smooth manner while the *minH* policy shows a big reduction in σ^* when the load goes from from 0.82 to 0.81. We may conclude that incorporating one or two FDLs may result in a significant cost reduction, as fewer WCs are needed. However, the results suggest that additional FDLs have little use as they affect the required number of FDLs in a less profound manner, especially for higher loads.

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