Performance Analysis of a Class of Randomly Addressed Polling Schemes for Wireless MAC Protocols *

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Abstract

This paper presents, evaluates and compares a family of Randomly Addressed Polling (RAP) schemes for wireless MAC protocols. These RAP schemes are collision resolution protocols implementing a decentralized from of polling. They can be used to resolve contention in wireless networks, in particular for best effort traffic. Two classes of RAP schemes are considered : single layer and multiple layer schemes. For each class we discuss three variants. Their throughput is evaluated and compared for varying load and burstiness of the offered traffic of the mobile stations using a matrix-analytical approach.

Keywords : Randomly Addressed Polling, Medium Access Control, Wireless Networking.

1 INTRODUCTION

Due to the rapid development of powerful high performance portable computers and other mobile devices, there is an increasing interest in wireless communication systems, in particular for Local Area Networks (e.g. in an office environment). These wireless LANs need to be connected in a seamless fashion to the fixed network. For these networks, the Asynchronous Transfer Mode (ATM) has been standardized as transfer mode supporting the Quality of Service (QoS) requirements of different service categories in an efficient way. In the study to define an ATM based transport architecture for an integrated

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services wireless network, the definition of an efficient Medium Access Control (MAC) protocol is of major importance.
This paper presents an analytical evaluation of a family of contention resolution schemes for wireless LANs, called *Randomly Addressed Polling* (RAP). We consider a cellular system, where a base station (BS) is located in the center of a cell in which a variable number of mobile stations (MS) are moving. Communication is divided according to the direction : the BS is capable of broadcasting information to all the MSs, referred to as *downlink communication*, while the MSs have to share the available bandwidth to send information towards the BS (i.e. *uplink communication*). This multiple access uplink channel needs a MAC protocol to arbitrate the access. Different proposals of MAC protocols for wireless ATM systems have been made (see e.g. [7], [8], [9], [10], etc.). Randomly Addressed Polling, a family of collision resolution protocols has been proposed to service best effort traffic (e.g. the Unspecified Bit Rate (UBR) ATM service category) in this context (see [3], [4], [5], [6]).
In this paper we present a unified approach to evaluate the throughput of a family of schemes belonging to this class. Based on the results obtained for the existing proposals, we propose an enhancement and show its influence on the system throughput.

2 RANDOMLY ADDRESSED POLLING SCHEMES

In this Section we describe the different protocols belonging to the RAP family and refer to the corresponding papers for more details.

2.1 Randomly Addressed Polling (RAP) and its Variants

The original *Randomly Addresses Polling (RAP)* protocol was introduced in [3] and can be seen as a decentralized form of polling [5]. The operation of RAP can be summerized as follows :

1. When a base station (BS) is ready to receive packets in the uplink direction it broadcasts a {READY} signal to all the mobile stations (MS) in its area.
2. Each active MS (i.e. an MS ready to transmit a packet) generates a random number between 1 and p, p being a protocol parameter. The active MSs transmit these numbers simultaneously using CDMA (by means of p orthogonal codes) or using FDMA (by means of p different frequencies).
3. After receiving these numbers, the BS polls the active mobiles by transmitting the received numbers one by one, and thus giving permission to use the uplink channel to transmit a packet. In case more than one MS has generated the same number, these MSs transmit a packet simultaneously and a collision occurs. Acknowledgements are used to inform the mobile(s) whether the transmission was successful or not.

4. Steps 1 to 3, referred to as a *polling cycle*, are repeated for all unsuccessfully polled MSs, until all active MSs have sent their packet successfully (a different signal is used in step one to indicate that not all collisions were resolved).

As in [13] we define the *Collision Resolution Cycle* (CRC) as the time needed to allow all active mobiles to have a successful transmission. Only MSs having a packet ready at the start of a CRC, participate in this resolution cycle.

Example : Suppose that 7 MSs have a packet to transmit at the moment that the BS broadcasts the {READY} signal and assume that the parameter p is set to 10. Let's say that during the first cycle number 1 is generated once, number 3 twice, number 7 three times and finally number 9 is selected once too. Clearly this results in 2 successful and 2 unsuccessful transmissions (see figure 1, where the unsuccessful transmissions are denoted as (long) white packets and the others are marked). Thus 5 MSs remain after the first cycle. Suppose now the during the second phase all mobiles but two generate a unique number (say 2, 3 and 9 are the unique numbers), then we have just one collision during cycle two. In figure 1 it is assumed that the two MSs involved in that collision have selected a different number during the third step/cycle (say number 3 and 5). Thus this CRC is composed of three cycles.

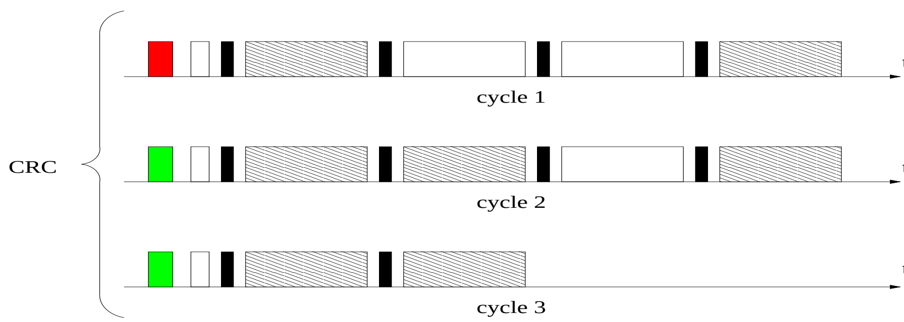

Figure 1 an example of RAP

In order to improve the performance of the RAP scheme, the following enhancement can be considered. The basic idea is to reuse in the first cycle of a CRC the number that was used to successfully transmit a packet in the previous CRC. When this number does not lead to a successful transmission in the first polling cycle of the current CRC, a new number will be used to resolve the contention. The following variants can be distinguished.

1. When an MS was able to transmit its previous packet *without any collision*

(i.e. in the first polling cycle of the previous CRC), it will use the same number in the next CRC. This version of RAP is called RAPO (optimized version of RAP).
2. In a slightly different version, called RAPO', an MS is allowed to use in (the first polling cycle of) the current CRC the number which led to a successful transmission during the previous CRC (even if this successful transmission was preceded by one or more collisions during the previous CRC).
3. Finally we introduce a new version, called RAPO+. The main design purpose of this new protocol variant will be given when we introduce GRAPO+, which is the multi-layered version of this scheme.
   If an MS was able to send a packet without any collision (i.e. the number of the first polling cycle was successful), this number will be used during (each first polling cycle) of all the future CRCs (as long as the MS remains active). This means that if a collision occurs during the first polling cycle of a future CRC, the MSs involved in the collision generate a new random number in this CRC to resolve the contention. However, in the next CRC the MSs use again the original number. This version is referred to as RAPO+.

Example : We continue with the previous example to demonstrate the differences between each of these protocol variants. Assume that all seven MSs that just participated in the scheme still have packets remaining. In the case of RAP all seven MSs will generate a new random number, for RAPO the two MSs that were successful during the first cycle (with number 1 and 9) will reuse these numbers (1 and 9) while the other five MSs generate a new random number. Thus these two MSs will never collide with each other when transmitting their packet for the first time. For the RAPO' protocol all 7 MSs will reuse their success number, that is 1, 2, 3, 5 and 9 are the numbers used. This will result in 2 collisions (on number 3 and 9) and 3 successes. The numbers used by the RAPO+ protocol do not only depend on the previous CRC but also on those before and thus it is impossible to determine them in this example.

As with the original ALOHA and CSMA protocols (see [12]) this scheme might suffer from instability. This has lead to the introduction of Group Randomly Addressed Polling (GRAP) schemes [4].

2.2 Group Randomly Addressed Polling (GRAP) and its variants

First we describe the modifications to RAP which have led to *Group Randomly Addressed Polling (GRAP)*. The main improvement is obtained by introducing a super frame structure consisting of G frames, also referred to as groups.

An MS that is active has to chose a random number to obtain a group and within each group the RAP protocol as defined above applies. This implies that contention is resolved completely within a group before the next group is dealt with. A CRC is again defined to be the time needed to allow all active MSs to have a successful transmission (i.e. the time needed to treat all groups).
A broadcast period can be provided between each of the G frames, allowing downlink traffic to be transmitted on regular basis.
In [4] it is shown that the stability and performance of RAP is improved by GRAP while keeping all desirable features of RAP. For more details on GRAP we refer to [5, 4].
As with the RAP scheme, the question rises whether the numbers (both group and number within a group) that lead to a successful transmission can be reused in later CRCs. This leads to a number of variants of the GRAP protocol.

1. In the GRAPO scheme, mobiles that were successfully polled during the first polling cycle of their corresponding group remain within the same group and use the same number within that group. Moreover, the number of groups (i.e. the number of frames in the super frame) is made dynamic. The latter characteristic is not considered in our analysis. For more details on GRAPO we refer to [11], where it is considered as one of the interesting proposals for wireless MAC protocols. However, it still has some disadvantages as is shown in the following example. Assume a GRAPO system with G groups and a new MS joins the polling scheme. According to the GRAPO protocol, this MS selects a group and a random number within that group. Clearly if all the other mobiles participating in the polling scheme have at that point in time a unique number within a group (obtained by means of the GRAPO protocol), one of them might loose it by colliding with the packet of the newly activated mobile. If so at least two mobiles will select a new group and a number for the first polling cycle of the next CRC. Again, as a consequence, other mobiles might loose their unique number within a group because of collision. Thus in many cases the participation of a new mobile in the polling scheme will cause a chain reaction and it may take some time before the scheme returns to be collision free.
2. To avoid this situation, we propose a modification of the GRAP scheme. A mobile obtains a group and a number within that group. If these numbers lead to a successful transmission, it continues using these numbers during the first polling cycle of each corresponding uplink period as long as it keeps on participating in the polling scheme. In this way we avoid a chain reaction. In addition the scheme will remain fair since we use RAP as the collision resolution scheme within each group. We call this modified scheme GRAPO+, since it is the multi-layer version of the RAPO+ scheme.

The RAPO' protocol introduced above has no useful extension to the multi-layer case. Indeed, if a collision within a group occurs, it is more useful to allow the MS to join another group in order to avoid collisions again.

3 PERFORMANCE EVALUATION OF RANDOMLY ADDRESSED POLLING SCHEMES

In this section we present an analytical evaluation of the different Random Addressed Polling schemes. Throughout this Section, we use the following assumptions and notations.
Each MS can be in two states, a *transmission state*, during which the MS participates in the polling scheme (i.e. it has a packet ready to transmit to the BS), and a *sleep state*, during which the MS is silent and generates no packets. The probability that an active MS goes to the sleep state at the end of a CRC is denoted by p_{OFF}, while the reverse transition (i.e. from sleep to transmission) is denoted by p_{ON}. The probability for an MS to be in the transmission state is then given by $q = \frac{p_{ON}}{p_{ON}+p_{OFF}}$. Furthermore we use the following notations.

- T_s is the time needed to send a packet upstream
- T_c is the time needed to detect a collision
- T_{poll} is the time needed to poll the MSs
- T_{over} is the time needed to sense the different carriers or codes
- τ is the propagation time in the cell.

We evaluate the performance of the single layer and the multiple layer classes separately. For the single layer we consider two models. In the first model, referred to as the *static model*, we assume that the probability p_{ON} and p_{OFF} are small, i.e. the MSs remain active/passive for a long period. For that case, the analysis is made for a fixed number of MS in the transmission state and the throughput is obtained by means of a weighted sum of these results. This means that the time the systems needs to return to a stable situation after one or more MSs become active is not considered. In the second model, referred to as the *dynamic model*, the transition from transmission state to sleep state and vice versa after each CRC, are taken into account, leading to more accurate results.
We have introduced the static model to be able to show that the RAPO and RAPO' single layer schemes perform very similar. In this way, we avoid considering the more complex (from a modeling point of view) scheme RAPO' in the more detailed dynamic model (see Section 4.1).

3.1 Single Layer: A Static Model

In this first Subsection we evaluate the single layer RAP family protocols through a static model. Consider a system with N mobiles. We assume that p_{ON} and p_{OFF} are small.

(a) Evaluation of RAP

In [3] it is shown that the mean time to transmit n packets is given by

$$T_{RAP}(n) = \sum_{i,j} P_n^p(i,j)[T_{over} + iT_{ins} + jT_{inc} + T_{RAP}(n-i)],$$

where $T_{ins} = T_{poll} + T_s + \tau$, $T_{inc} = T_{poll} + T_c + \tau$ and $P_n^p(i,j)$ is the probability of i successfully transmitted packets and j colliding numbers (in a set of p) given n active mobiles. Using the result for $T_{RAP}(n)$, it is possible to compute the throughput as

$$S_{RAP}^{stat} = T_s * N * q * \left[\sum_{n=0}^{N} \binom{N}{n} q^n (1-q)^{N-n} T_{RAP}(n) \right]^{-1}.$$

(b) Evaluation of RAPO'

Assume that n mobiles are participating in the polling scheme all the time. The system is said to be in state $s = (s_1, \ldots, s_k)$ with $k \leq n-1$ at the start of a CRC, if in the previous CRC there were k polling cycles in which there were successfully transmitted packet(s) (the other polling cycles of this CRC are of no importance in our model); s_i denotes the number of packets that was transmitted successfully during the i-th of those k polling cycles ($\sum_{i=1}^k s_i = n$). This state space is further reduced by defining the equivalence relation $x \sim_P y$ if x is a permutation of y and we will represent each class by its descending representative.

Define $P^p_{(s_1,\ldots,s_k)}(i,j)$ as the probability of having i unique numbers and j colliding numbers (in a set of p) during the first polling cycle when being in the state $(s_1, \ldots, s_k)$ at the start of a CRC. This allows us to calculate the average length of a CRC for this model as

$$T(n) = \sum_s P(s) \sum_{i,j} P^p_{(s_1,\ldots,s_k)}(i,j)((T_{over} + iT_{ins} + jT_{inc} + T_{RAP}(n-i)),$$

with T_{ins} and T_{inc} as above, $T_{RAP}(j)$ the mean time needed by the RAP protocol to transmit j packets computed in the previous section, and $P(s)$ the probability of being in state $s = (s_1, \ldots, s_k)$. Using induction on the number of sets we can calculate $P^p_{(s_1,\ldots,s_m)}(i,j)$ as follows:

$$P^p_{(s_1,\ldots,s_m)}(i,j) = \sum_{k,l} \frac{C^k_{x_1} C^l_{x_2} C^{p-k-l}_{s_m - x_1 - x_2}}{C^p_{s_m}} P^p_{(s_1,\ldots,s_{m-1})}(k,l)$$

where $x_1 = j-l$, $x_2 = k+s_n-2(j-l)-i$ and C_k^n is the number of ways to choose k numbers within a set of n. The probability $P(s)$, i.e. the probability that the system is in state $s = (s_1, \ldots, s_n)$, is obtained by considering a Markov chain of which the transition matrix is computed using $P^p_{(s_1,\ldots,s_m)}(i,j)$.
The throughput of this model with n active station equals $T_s * M/T(n)$.
As mentioned earlier we only consider long transmission periods (i.e. large q values) neglecting the time interval needed to return to stability for the new number of sources in the transmit phase. Since $q^n(1-q)^{N-n}$ is the probability of having n MS in a transmission state, $\left[\sum_{n=0}^{N} \begin{pmatrix} N \\ n \end{pmatrix} q^n(1-q)^{N-n}T(n)\right]$ is the mean time needed to finish a CRC. Hence, since the time needed to send all packets is given by $T_s * N * q$, the system throughput is given by

$$S^{stat}_{RAPO'} = T_s * N * q * \left[\sum_{n=0}^{N} \begin{pmatrix} N \\ n \end{pmatrix} q^n(1-q)^{N-n}T(n)\right]^{-1}. \qquad (1)$$

(c) Evaluation of RAPO

It is clear that this variant is much easier to evaluate than RAPO', since only numbers which were successful in the first polling cycle of a CRC are allowed to be reused. Again we start with the simplified model of n mobiles sending traffic all the time. Now the state space is $\{0, 1, \ldots, p\}$ and state j corresponds to the situation in which j of the n mobiles have received a number (which is unique among all mobiles) which resulted from the last CRC. The probability of having i successes and j collisions in the first polling cycle given that we were in state k, referred to as $P_k^p(i,j)$, can be obtained as

$$P_k^p(i,j) = P^p_{(k,\underbrace{1,\ldots,1}_{n-k})}(i,j),$$

using the notations introduced in the evaluation of RAPO'. The remaining part of the evaluation follows the same reasoning as for RAPO', resulting in an expression for the throughput S^{stat}_{RAPO}.

(d) Evaluation of RAPO+

Assuming that n terminals participate in the scheme all the time it is obvious that all p numbers will be associated with a unique mobile (if n is larger than p, otherwise only n numbers are). As a consequence the throughput S^{stat}_{RAPO+} can be found using equation 1 but now $T(n)$ is computed as

$$T(n) = \sum_{i,j} P_k^p(i,j)[T_{over} + iT_{ins} + jT_{inc} + T_{RAP}(n-i)],$$

where k matches $\min(n,p)$ and $P_k^p(i,j)$ is defined in the previous section.

3.2 Single Layer: a Dynamic Model

The main drawback of this static model is that if the number of mobiles in the 'transmit' phase changes, a certain period of time must pass before the steady state associated with that number of active mobiles is reached. As a consequence it cannot be used in case of short transmit periods. In this section we present a model for which this assumption on the duration of the transmit phase is not needed any longer.
Again we assume that there are N mobiles within the area, each of them being in the 'transmit' or 'sleep' phase. Observing the process at the beginning of each CRC, the total mobile traffic consists of a superposition of ON/OFF traffic sources. In [1], it has been shown that this traffic can be modeled by means of a Discrete-time Batch Markovian Arrival Process (D-BMAP) (for more details see [1]).
Notice that reuse of a successful number only applies within a transmit period. When returning from a sleep period, the MS uses a random number in the first CRC.

(a) Evaluation of RAP

Since all mobiles only use random numbers during each CRC and as the probability of having n active mobiles is equal in both the static and dynamic model, the throughput in both systems is the same.

(b) Evaluation of RAPO

To analyse the RAPO scheme, we use a Markovian model, where the system is said to be in state (n, j) if there are n mobiles in transmit phase and j of them have obtained a number (which is unique among all mobiles) during the last CRC. Thus the number of states equals

$$(N+1) + \frac{\min(N,p) * (1+\min(N,p))}{2} + (N-p) * p * 1_{\{N>p\}}, \tag{2}$$

where 1_A is the characteristic function of A. Using a similar notation as in Section 3.1c, we can compute $P^p_{(n,k)}(i,j)$, being the probability of having i unique numbers out of p and j collision numbers during the first polling cycle of a CRC, given that at the start of this CRC the state is (n, k). Using the values for $P^p_{(n,k)}(i,j)$, it is possible to define the matrix which governs the state transitions between starts of consecutive CRCs. Solving for the steady state vector, we obtain $P(s)$, being the probability that the system is in state $s = (n, k)$ at the start of a CRC. The mean length of a CRC for a system with N mobiles is given by

$$T(N) = \sum_s P(s) \sum_{i,j} P^p_s(i,j)[T_{over} + iT_{ins} + jT_{inc} + T_{RAP}(n-i)], \tag{3}$$

with $s = (n, k)$. The throughput of the RAPO scheme is then given by

$$S^{dyn}_{RAPO} = \frac{N * q}{T(N)}.$$

(c) Evaluation of RAPO+

The analysis of the RAPO+ scheme is similar to the one used for the RAPO scheme. The computation of the transition matrix is simpler, as a number which has been successful in a CRC will be reused in all future CRCs (as long as the corresponding mobile remains active).

3.3 Multiple Layers

We use the same notation as for the single layer analysis.

(a) Evaluation of GRAP

The GRAP protocol can be evaluated as in [4]. An alternative approach is based on the observation that the throughput in each group is expected to be equal. Hence, we can tag a specific group and calculate its throughput. For the model above this results in the following formula

$$S_{GRAP} = \frac{N/G * q * T_s}{\sum_{i=0}^{N} \sum_{j=0}^{i} C_i^N C_j^i q^i (1-q)^{N-i} (1/G)^j (1-1/G)^{i-j} T_{RAP}(j)}, \tag{4}$$

where q is the probability that a mobile is in the transmit phase C_k^n is the number of ways to choose k numbers within a set of n and as before, $T_{RAP}(i)$ is the time needed by the RAP protocol to transmit i packets.

(b) Evaluation of GRAPO+

As with GRAP we compute the throughput of a tagged group. The other $G-1$ groups will be referred to as the background groups. Consider a Markov chain, where the system is described by the vector $s = (n, s_b, s_t)$ at the end of the CRC cycle corresponding with the last group, where

- n is the number of mobiles in the transmit phase,
- s_b of those mobiles have obtained a unique number within a background group
- s_t mobiles have obtained a unique number in the tagged group.

Thus $n \leq N$, $s_t \leq p$, $s_b \leq (G-1) * p$ and $s_t + s_b \leq n$. Once we know the probability $P(s)$ of being in state s the throughput is given by

$$S_{GRAPO+} = \frac{N * q/G * T_s}{\sum_s P(s) \sum_{n'=0}^{n-s_t-s_b} P_t(n') \sum_{i,j} P^p_{(n'+s_t,s_t)}(i,j) \mathcal{F}(i,j,n',s_t)}, \tag{5}$$

with $P_t(n')$ the probability that n' of the $n - s_t - s_b$ mobiles select the tagged group, $P^p_{(n'+s_t,s_t)}(i,j)$ as defined in section 3.2b, and

$$\mathcal{F}(i,j,n',s_t) = T_{over} + i * T_{ins} + j * T_{inc} + T_{RAP}(n' + s_t - i). \qquad (6)$$

The main difficulty consists of calculating the transition probabilities in an efficient way. Denote by $P(s,s')$ the transition probability from state $s = (n, s_b, s_t)$ to state $s' = (n', s'_b, s'_t)$. We sketch a method to obtain these values. First compute

$$C^{(n,s_b,s_t)}_{(s_1,s_2)} = \sum_{i=[k]^+}^{n} \left(\sum_j P^{G*p}_{(n-s_b-s_t,0)}(i,j) \right) \frac{C^{s_b+s_t}_{i-k} C^{G*p-s_b-s_t}_{k}}{C^{G*p}_{i}} \frac{C^{p-s_t}_{s_2-s_t} C^{(G-1)*p-s_b}_{k-s_2+s_t}}{C^{G*p-s_t-s_b}_{k}}, \qquad (7)$$

with $k = s_1 + s_2 - s_b - s_t$ and $[x]^+ = \max(0,x)$. Notice that the value above represents the probability that the first polling cycles of all groups results in s_1 unique numbers within the background groups and s_2 in the tagged group. Taking into account that some of these mobiles may switch to the sleep phase, we obtain the following expression for $P(s,s')$

$$P(s,s') = \sum_{s_1 \geq s'_b, s_2 \geq s'_t} C^{(n,s_b,s_t)}_{(s_1,s_2)} B^{s_1}_{s_1-s'_b}(q_1) B^{s_2}_{s_2-s'_t}(q_1) D_{s_1,s_2}(s,s') \qquad (8)$$

$$D_{s_1,s_2}(s,s') = \sum_{i=0}^{n-s_1-s_2} B^{n-s_1-s_2}_{i}(q_1) B^{N-n}_{n'-n+k'+i}(q_2), \qquad (9)$$

with $k' = s_1 + s_2 - s'_b - s'_t$, $q_1 = p_{OFF}$, $q_2 = p_{ON}$ and $B^n_k(p)$ being the probability of k successes in a binomial distribution with parameters (n,p).

(c) Evaluation of GRAPO

Let us now focus on the evaluation of the GRAPO protocol. Opposed to the GRAPO+ scheme, a mobile may now also loose the "success" number it obtained in previous CRCs due to a collision with one or more mobiles that did not obtain a number yet. However, in view of the similarities between the two schemes, the state space description remains unchanged, as well as formulas (5),(6),(8) and (9). The new expression for equation (7) must also take the collisions into account, resulting in

$$C^{(n,s_b,s_t)}_{(s_1,s_2)} = \sum_{i,j} \sum_{c=[-k]^+}^{s_b+s_t} \sum_{c_t=0}^{c} P^{G*p}_{(n-s_t-s_b,0)}(i,j) \cdot \frac{C^{Gp-s_b-s_t}_{i+j-c} C^{s_t}_{c_t} C^{s_b}_{c-c_t}}{C^{Gp}_{i+j}} \frac{C^{i+j-c}_{k+c} C^{c}_{i-k-c}}{C^{i+j}_{i}} \frac{C^{(G-1)p-s_b}_{s_1-s_b+(c-c_t)} C^{p-s_t}_{s_2-s_t+c_t}}{C^{Gp-s_t-s_b}_{k+c}},$$

with $k = s_1 + s_2 - s_t - s_b$ and $[x]^+ = \max(0, x)$. This expression contains three parts. The first part represents the probability that c mobiles loose their number by colliding and c_t of them where in the tagged group. The second part gives the probability that the remaining $i + j - c$ numbers amount to the correct number $(s_1 + s_2)$ of mobiles with a unique number and finally part three gives the probability by which the mobiles that just obtained a number are divided between the tagged group and the background group.

4 NUMERICAL RESULTS

4.1 The Single Layer: RAPO vs. RAPO'

In this first numerical example we compare the RAPO and RAPO' protocol using the static model. Let $N = 12, p = 6, T_s = T_c = 1, \tau = 0.001, T_{poll} = 0.01$ and $T_{over} = 0.06$. Figure 2 shows that both protocols outperform RAP in case of a low to medium load and perform slightly worse for high load situations. This can be explained by the fact that in case of more than p active mobiles, collisions in which a high number of mobiles are involved is preferable, such that the other mobiles can use the remaining set of numbers to send their packet (we give a more formal explanation of this phenomenon at the end of this section). Moreover, both the RAPO' and RAPO protocol perform very similar, more specific, RAPO' performs slightly better than RAPO for low to medium loads while the contrary is true for high loads. Still we can approximate the analytically less tractable RAPO' protocol by the RAPO scheme. Since more bursty mobiles are not expected to influence these last results in what follows we only evaluate the RAPO protocol for the dynamic model and assume that the RAPO' scheme performs likewise.

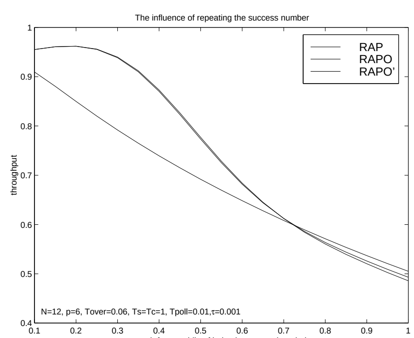

Figure 2 Comparison between the RAPO and RAPO' protocol for variable load

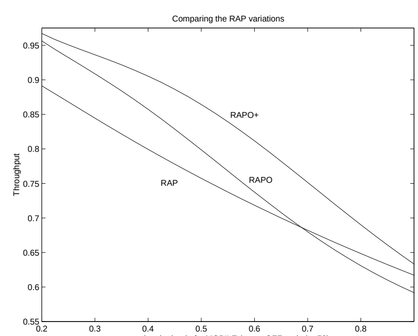

Figure 3 Comparison between RAP, RAPO and RAPO+ for variable load

We now present some analytical arguments that suggest that the relationship between the RAPO and RAP curve in Figure 2 remains valid for different (larger) values of p and n. Define the following set of functions $f_i(n)$ on the interval $[i, +\infty[$

$$f_i(n) = \frac{(1-1/p)^{n-i}}{p-1}[i^2 - i(n+1) + np], \quad 0 \leq i \leq p.$$

It is easy to shown that $f_i(n)$ is the expected number of unique numbers during the first polling cycle of a CRC in the RAPO protocol, given that i mobiles had a unique number and n mobiles participate in the scheme. Also notice that $f_0(n)$ equals the expected number of unique numbers during the first polling cycle for the RAP protocol. In Appendix 1 it is shown that $f_i(n) - f_0(n)$ can be written as

$$f_i(n) - f_0(n) = \frac{(1-1/p)^n}{p-1} * (n * c_1 + c_2),$$

with $c_2 \geq 0$ and $c_1 \leq 0$. It is easily seen that $f_i(i) - f_0(i) \geq 0$; thus $f_i - f_0$ is positive in i and because of the equation above, remains so as long as $-n * c_1$ is smaller than c_2 (it can be shown that this is still the case for $n = p$). For these values of n the expected number of successful transmissions during the first polling cycle will be higher when using RAPO than when using RAP. As n is further increased $f_i - f_0$ will become negative thus RAP is expected to have more unique numbers for the first polling cycle. As n goes to infinity the value of $f_i - f_0$ approaches zero. Given the strong relation between the expected number of unique numbers and the throughput characteristics we have a clear indication that the result observed in Figure 2 is not restricted to the specific values of the different parameters used.

4.2 The Single Layer: A comparison between the different protocols and the influence of the burstiness of the traffic on these results

In this second example we will use the more detailed dynamic model with $N = 7$, $p = 4$ and the other parameters chosen as in the first example. Again the load is varied by increasing the mean transmit period while keeping the mean sleep period equal to 50. As can be seen in Figure 3 the RAPO+ protocol clearly outperforms the other two protocols even if the load approaches one. Other numerical experiments have shown that when choosing N about 3 times (or more) the value of p, the RAP protocol performs better than the other variants (if $N = 8, p = 3$ then RAP has a higher throughput then RAPO+ once the probability of being in the transmit phase is larger then 0.525). If this probability is further increased the performance of RAPO+ will drop below the one of RAPO, though this situation is quite unlikely if we use the corresponding GRAPO+ protocol.

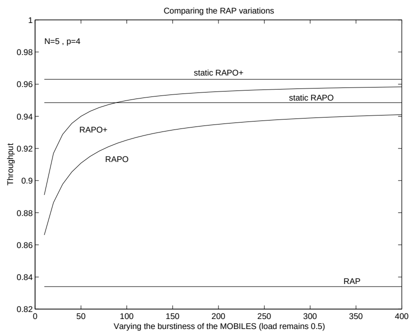

Figure 4 Comparison between the RAP , RAPO and RAPO+ protocol for variable burstiness of the offered traffic and low load

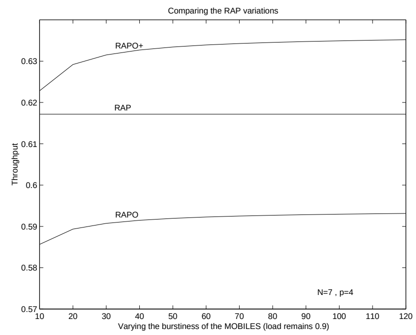

Figure 5 Comparison between the RAP, RAPO and RAPO+ protocol for variable burstiness of the offered traffic and high load

Before investigating the influence of the burstiness on this result we give a more analytical explanation why a similar relationship between the curves corresponding to the different protocols is to be expected for other values of the system parameters. First, notice that the argument used to explain the interaction between the RAP and RAPO curves is also valid for the relation between RAP and RAPO+. If we want to establish a similar result for RAPO and RAPO+ we have to study $f_j - f_i$ for $i \leq j$ since (for our dynamic model) the set of mobiles with a unique number is expected to be larger if we use RAPO+. In the Appendix it is shown that $f_j - f_i$ can be written in a similar form as $f_i - f_0$ and given that $f_j(j) - f_i(j)$ is positive (which is easy to show for $i \leq j$) we have a similar relationship between RAPO+ and RAPO as we had between RAPO and RAP.

It is expected that the gains obtained when using the protocols RAPO and RAPO+, with respect to the classical RAP (in case of low to medium load) will increase together with the burst size of the offered traffic, an idea which is confirmed by Figure 4 where both curves approach the static model for large transmit periods. We also notice that according to the design purpose of RAPO+, the curve corresponding with the RAPO+ protocol stabilizes more quickly. Figure 5 shows a similar result for high load (=0.9). Remark that, according to the result obtained in Figure 3, RAP performs better than RAPO for this high load.

4.3 Multi-Layered Variants of the RAP Protocol

In this section we study the influence of the multi-layer structure of GRAP, GRAPO and GRAPO+. As before we look at the influence of the load and burstiness of the offered traffic on the throughput. Moreover, these results are compared with the corresponding single layer results. For the multi-layer case, we let $G = 3$, $N = 15$ and $p = 4$, while for the single layer case we let $p = 12$. We let the remaining parameters be as in the first example, except in the single layer case, we let $T_{over} = 0.18$ as servicing the different carries or codes takes more time. When investigating the influence of the load, we let $p_{ON} = 1/20$, while for the study of the influence of the burstiness of the offered traffic, we let $p_{ON} = p_{OFF}$.

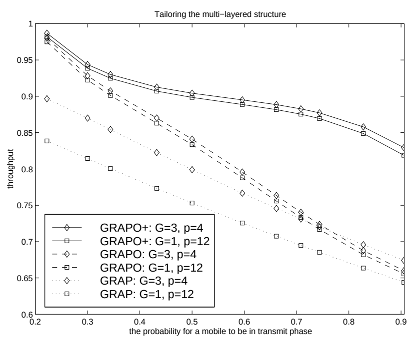

Figure 6 Comparison between the different protocols for variable probability to be in transmit phase

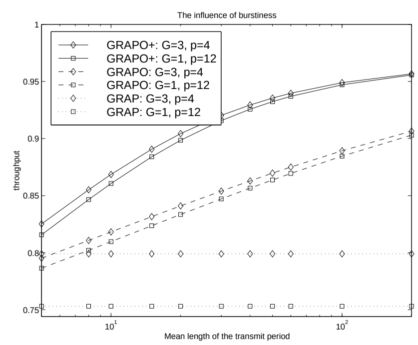

Figure 7 Comparison between the different protocols for variable burstiness of the offered traffic

Figure 6 shows that GRAPO+ clearly outperforms GRAP and GRAPO. The performance of GRAP is better than GRAPO as the length of the transmit period becomes large. Moreover the impact of introducing multiple layers on the system throughput seems much smaller on both the GRAPO and the GRAPO+ protocol than on the GRAP protocol. This last assertion is confirmed when changing the burstiness of the offered traffic (see figure 7). Moreover, as the burst size grows the influence of the multiple layer structure decreases, due to the fact that the only difference between multiple layers and a single layer lies within the collision resolution strategy.

5 CONCLUSIONS

In this paper we have introduced a family of protocols obtained as variants from the Randomly Addressed Polling scheme. We have considered two classes, namely single layer and multiple layer schemes. This results into the following variants : (G)RAP, (G)RAPO, RAPO' and (G)RAPO+. The variants (G)RAP and (G)RAPO were defined in [3] and [4]. This paper studies the impact of the different variants on the throughput for variable load and burstiness of the offered traffic, using a matrix analytical approach. Numerical results have shown that no significant differences are to be expected between the RAPO and RAPO' scheme. The (G)RAPO+ protocol realizes a considerable improvement on the throughput, especially for bursty mobile traffic. Finally, the impact of introducing multiple layers on the performance is smaller in case of the GRAPO+ protocol than for the GRAP scheme, and depends on the burstiness of the offered traffic.

6 APPENDIX

6.1 $f_i - f_0$

We now prove that $f_i - f_0$ can be written as

$$f_i(n) - f_0(n) = \frac{(1-1/p)^n}{p-1} * (n * c_1 + c_2),$$

with $c_2 \geq 0$ and $c_1 \leq 0$. Some elementary calculations show that c_2 matches

$$c_2 = i(i-1)\left(\frac{p}{p-1}\right)^i$$

which is clearly a positive value since $i \geq 1$. It remains to show that c_1 which matches the formula below is negative

$$c_1 = (p-i)\left(\frac{p}{p-1}\right)^i - p$$

for $1 < p$ and $1 \leq i \leq p$. Since the case of $i = p$ is trivial, we will prove by induction on i that

$$\left(\frac{p}{p-1}\right)^i \leq \frac{p}{p-i}.$$

For $i = 1$ we have an equality thus by induction we get

$$\left(\frac{p}{p-1}\right)^i \leq \frac{p^2}{p^2 - (ip - i + 1)} \leq \frac{p}{p-i}$$

using the fact that $i \geq 1$ which proves the equation.

6.2 $f_j - f_i$ with $i \leq j$

In this section we extend the result of the previous section, i.e. we show that $f_j - f_i$ for $0 \leq i \leq j \leq p$ can be written as:

$$f_j(n) - f_i(n) = \frac{(1 - 1/p)^{n+i}}{p - 1} * (n * d_1 + d_2)$$

with $d_1 \leq 0$ and $d_2 \geq 0$ and independent of n. As before the only non-triviality lies in proving that d_1 is negative. It is easily shown that d_1 equals

$$d_1 = (p - j) * \left(\frac{p}{p-1}\right)^{j-i} - (p - i).$$

Clearly this is negative or zero for $j = p$ and $j = i$. Thereby it is sufficient to shown that

$$\left(\frac{p}{p-1}\right)^{j-i} \leq \frac{p - i}{p - j}$$

for $0 \leq i < j \leq p - 1$. This can be done as follows using the result of the previous section for the first inequality

$$\left(\frac{p}{p-1}\right)^{j-i} \leq \frac{p}{p - (j - i)} = \frac{(p - i) + i}{(p - j) + i} \leq \frac{p - i}{p - j}$$

since $(p - i)/(p - j) \geq 1$ and if the fraction $a/b \geq 1$ then $a/b \geq (a + i)/(b + i)$ for i positive.

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